Rješenje:

Odredimi determinante sustava

$$D = \begin{vmatrix} 1 & m+1 \\ m+1 & 4m \end{vmatrix} = 4m - (m+1)^2 = -(m-1)^2$$

$$D_x = \begin{vmatrix} m & m+1 \\ m+1 & 4m \end{vmatrix} = 4m^2 - (m+1)^2 = 3m^2 - 2m - 1 = (m-1)(3m+1)$$

$$D_y = \begin{vmatrix} 1 & m \\ m+1 & m+1 \end{vmatrix} = m + 1 - m^2 - m = 1 - m^2$$

1) Za $D = 0 \Rightarrow -(m-1)^2 \Rightarrow m = 1$. No tada su: $D_x = 0$ i $D_y = 0$, a to znači da je sustav **neodređen**.

2) Za $m \neq 1$ je determinanta sustava $D \neq 0$, pa sustav ima jedninstveno rješenje dato sa

$$x = \frac{D_x}{D} = \frac{(m-1)(3m+1)}{-(m-1)^2} = -\frac{(3m+1)}{m-1}$$

$$y = \frac{D_y}{D} = \frac{1-m^2}{-(m-1)^2} = \frac{m+1}{m-1}$$

Sada je $x+y = \frac{-3m-1}{m-1} + \frac{m+1}{m-1} = \frac{-2m}{m-1}$. Zbog traženog uvjeta $|x+y| \leq 1$ imamo nejednadžbu

$$\left|\frac{-2m}{m-1}\right| \leq 1$$

odnosno ($|a| < b \Leftrightarrow -b < a < b$) nejednadžbu

$$-1 \leq \frac{-2m}{m-1} \leq 1$$

iz koje slijede dvije nejednadžbe

a)

$$-1 \leq \frac{-2m}{m-1} \qquad / \cdot (-1)$$

$$\frac{2m}{m-1} \leq 1$$

$$\frac{m+1}{m-1} \leq 0$$

$\mathbf{a}_1$: $m+1 \geq 0$, $m-1 < 0$	$\mathbf{a}_2$: $m+1 \leq 0$, $m-1 > 0$
$m \geq -1$	$m \leq -1$
$m < 1$	$m > 1$
$m \in [-1, 1\rangle$	$\varnothing$

$R_a : m \in [-1, 1\rangle$

b)

$$\frac{-2m}{m-1} \leq 1 \qquad / \cdot (-1)$$

$$\frac{2m}{m-1} + 1 \geq 0$$

$$\frac{3m-1}{m-1} \geq 0$$

$\mathbf{b}_1$: $3m-1 \geq 0$, $m-1 > 0$	$\mathbf{b}_2$: $3m-1 \leq 0$, $m-1 < 0$
$m \geq \frac{1}{3}$	$m \leq \frac{1}{3}$
$m > 1$	$m < 1$
$m \in [1, \infty\rangle$	$m \in \left\langle -\infty, \frac{1}{3}\right]$

$R_b : m \in \left\langle -\infty, \frac{1}{3}\right] \cup [1, \infty\rangle$

Rješenje je

$$R = R_a \cap R_b = \left[-1, \frac{1}{3}\right]$$