

Adicijski teoremi - Formule

1. Kosinus i sinus zbroja i razlike¹

$$\begin{aligned}\cos(s+t) &= \cos s \cos t - \sin s \sin t \\ \cos(s-t) &= \cos s \cos t + \sin s \sin t \\ \sin(s+t) &= \sin s \cos t + \cos s \sin t \\ \sin(s-t) &= \sin s \cos t - \cos s \sin t\end{aligned}$$

2. Formule redukcije za sinus i kosinus funkcije

$$\begin{array}{ll}\cos\left(\frac{\pi}{2}-t\right)=\sin t & \sin\left(\frac{\pi}{2}-t\right)=\cos t \\ \cos\left(\frac{\pi}{2}+t\right)=-\sin t & \sin\left(\frac{\pi}{2}+t\right)=\cos t \\ \cos(\pi-t)=-\cos t & \sin(\pi-t)=\sin t \\ \cos(\pi+t)=-\cos t & \sin(\pi+t)=-\sin t\end{array}$$

3. Tangens i kotangens zbroja i razlike funkcija

$$\begin{aligned}\tan(s+t) &= \frac{\tan s + \tan t}{1 - \tan s \cdot \tan t} \\ \tan(s-t) &= \frac{\tan s - \tan t}{1 + \tan s \cdot \tan t} \\ \operatorname{ctg}(s+t) &= \frac{\operatorname{ctg} s \cdot \operatorname{ctg} t - 1}{\operatorname{ctg} s + \operatorname{ctg} t} \\ \operatorname{ctg}(s-t) &= \frac{\operatorname{ctg} s \cdot \operatorname{ctg} t + 1}{\operatorname{ctg} s - \operatorname{ctg} t}\end{aligned}$$

4. Formule redukcije za tangens i kotangens

$$\begin{array}{ll}\tan\left(\frac{\pi}{2}-t\right)=\operatorname{ctg} t & \operatorname{ctg}\left(\frac{\pi}{2}-t\right)=\tan t \\ \tan\left(\frac{\pi}{2}+t\right)=-\operatorname{ctg} t & \operatorname{ctg}\left(\frac{\pi}{2}+t\right)=-\tan t\end{array}$$

5. Trigonometrijske funkcije dvostrukog kuta

$$\begin{aligned}\sin 2\alpha &= 2 \sin \alpha \cos \alpha \\ \cos 2\alpha &= \cos^2 \alpha - \sin^2 \alpha \\ \tan 2\alpha &= \frac{2 \tan \alpha}{1 - \tan^2 \alpha} \\ \operatorname{ctg} 2\alpha &= \frac{\operatorname{ctg}^2 \alpha - 1}{2 \operatorname{ctg} \alpha}\end{aligned}$$

¹Ivica Đuzel, prof. matematike i informatike

6. Još dvije korisne formule

$$\sin^2 \alpha = \frac{1 - \cos \alpha}{2}$$
$$\cos^2 \alpha = \frac{1 + \cos \alpha}{2}$$

7. Sinus i kosinus polovičnog kuta

$$\sin \frac{\alpha}{2} = \sqrt{\frac{1 - \cos \alpha}{2}}$$
$$\cos \frac{\alpha}{2} = \sqrt{\frac{1 + \cos \alpha}{2}}$$

8. Univerzalna zamjena

$$\sin \alpha = \frac{2 \tan \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}} \quad \cos \alpha = \frac{1 - \tan \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}} \quad \tan \alpha = \frac{2 \tan \frac{\alpha}{2}}{1 - \tan^2 \frac{\alpha}{2}}$$

9. Transformacija umnoška u zbroj

$$\sin x \cos y = \frac{1}{2} [\sin(x+y) + \sin(x-y)]$$
$$\cos x \sin y = \frac{1}{2} [\sin(x+y) - \sin(x-y)]$$
$$\cos x \cos y = \frac{1}{2} [\cos(x+y) + \cos(x-y)]$$
$$\sin x \sin y = \frac{1}{2} [\cos(x-y) - \cos(x+y)]$$

10. Transformacija zbroja u umnožak²

$$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$
$$\sin \alpha - \sin \beta = 2 \cos \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$
$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$
$$\cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

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